signal The difference between the observed and calculated values were probably due to errors in predicting $\sigma^* U^*$ The aim of the experiment was to demonstrate that the instrument responds to the plasma flow; this was demonstrated

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# Laminar Boundary-Layer Development on Yawed Cone


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If an integral method[1] is used in investigating the supersonic three-dimensional boundary layer on a yawed cone, the boundary-layer development in the direction along the cone generator is needed Because of the conical configuration, Blasius-type parabolic similarity may be assumed[2] to exist in that direction, giving the following expression as the boundary-layer thickness:

$$\Delta = K(\mu\, x/\rho\, u\,)^{1/2} \tag{1}$$

where

$\Delta$ = boundary-layer thickness in terms of transformed coordinate $Y$

$Y = \int \frac{\rho}{\rho_e}\, dy$

$x$ = distance along the generator of a cone (see Fig 1)

$e$ = subscript for the outer edge of boundary layer

$K$ = proportionality factor

This note is to show how an expression for $K$ can be obtained in terms of flow parameters for the case in which third-degree polynominals are assumed for the profiles in the boundary layer as follows:[2]

$$u/u_e = (3 - 2\eta)\eta^2 + a\eta(1 - 2\eta + \eta^2) \tag{2}$$

$$w/u_e = (3 - 2\eta)\eta^2(w\,/u\,) + b\eta(1 - 2\eta + \eta^2) \tag{3}$$

$$(H - H\,)/(H_0 - H\,) = (3 - 2\eta)\eta^2 + c\eta(1 - 2\eta + \eta^2) \tag{4}$$

where

$u$ = longitudinal velocity

$w$ = circumferential velocity

$H$ = total enthalpy

$\eta$ = $Y/\Delta$

$a,b,c$ = parameters

$s$ = subscript for surface

$0$ = subscript for freestream stagnation

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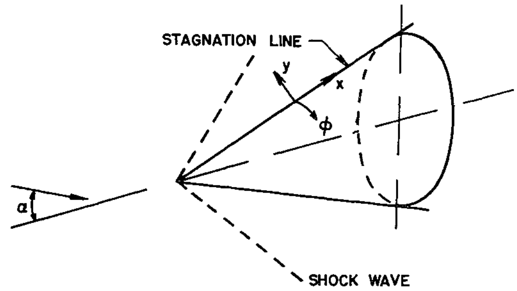


**Fig 1 Coordinate system for circular cone at an angle of attack**

The momentum equation in circumferential direction when applied at the surface is

$$\frac{dp}{d\phi} = x\beta\left[\frac{\partial}{\partial y}\left(\mu\,\frac{\partial w}{\partial y}\right)\right] \tag{5}$$

where

$\beta$ = sine of cone semivertex angle

$\phi$ = circumferential angle (see Fig 1)

Combining Eqs (3) and (5), we have

$$\frac{dp}{d\phi} = xu\,\beta\left\{\frac{\rho/\rho_e}{\Delta^2}\left[b\,\frac{\partial}{\partial\eta}\left(\mu\,\frac{\rho}{\rho}\right) + \mu\left(\frac{\rho}{\rho}\right)\left(6\left(\frac{w_e}{u}\right) - 4b\right)\right]\right\} \tag{6}$$

If the temperature-viscosity relation is assumed to be that of Chapman-Robeson type, i e,

$$\mu/\mu_e = C\; T/T_e \tag{7}$$

then the first term in the bracket of Eq (6) vanishes, and the expression of $dp/d\phi$ can be simplified as

$$\frac{dp}{d\phi} = x\beta u_e\left\{\left[\frac{\rho/\rho_e}{\Delta^2}\right]\left[\frac{\mu\rho}{\rho_e}\right]\left[6\left(\frac{w_e}{u_e}\right) - 4b\right]\right\} \tag{8}$$

Introducing $K$ in Eq (8) by using Eq (1), we have

$$K^2 = \beta u_e^2\rho\; C\left[\frac{6(w_e/u_e) - 4b}{dp/d\phi}\right] \tag{9}$$

$dp/d\phi$ can be determined from the flow conditions at the outer edge of boundary layer as follows: from the momentum equation of the inviscid flow, the following expression can be derived:

$$\frac{dp}{d\phi} = -\left[\rho\;\;u^{\,2}\left(\frac{w_e}{u_e}\right) + \rho\, u\,\left(\frac{w}{u}\right)^2\left(\frac{\partial u}{\partial\phi}\right) + \beta\rho\;\;u^{\,2}\left(\frac{w_e}{u_e}\right)\right] \tag{10}$$

Combining Eqs (9) and (10) we have

$$K^2 = \frac{\beta u^{\,2}\left[4b - 6\left(\frac{w_e}{u}\right)\right]\left[\frac{\rho\, C_s}{\rho}\right]}{\left[\left(\frac{w}{u}\right)u^{\,2}\,\frac{\partial(w_e/u\,)}{\partial\phi} + \left(\frac{w_e}{u}\right)^2 u\,\left(\frac{\partial u_e}{\partial\phi}\right) + \beta\left(\frac{w_e}{u}\right)u^{\,2}\right]} \tag{11}$$

It is thus seen that $K$ can be expressed in terms of the external flow parameters and the profile parameter of the circumferential velocity $b$

By using Eq (11) and the three boundary-layer equations of a yawed cone, the value of $K$ can be determined with the three profile parameters ($a$, $b$, and $c$) Since the shear parameter $f_w''$ has been obtained analytically for stagnation line, the following expression can be used to check with the values that can be obtained by using the present method:

$$f_w'' = (aC/K^2)/3 \tag{12}$$

The application of Eq (11) in finding the three-dimensional supersonic laminar boundary layer characteristics has been studied by obtaining results for a specific example for a range of circumferential angle of 90° A numerical method by using the digital computer was developed

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# Normal Shock Location in Underexpanded Gas and Gas-Particle Jets


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ONE of the important parameters pertinent to supersonic jets issuing from underexpanded nozzles is the distance from the nozzle exit plane to the normal shock wave existing in such flows This note presents results of experimental determination of this distance for both gas-only and gas-particle plumes The gas-only experiments were conducted in order to determine the effect of gas specific heat ratio on the normal shock position; this was accomplished by using various gases with different specific heat ratios Plumes containing macroscopic (micron-sized) condensed-phase particles also are of interest because of their relation to metallized solid propellant rocket exhausts Thus, experiments were also performed to determine the location of the normal shock as a function of particle loading in gas-particle jets, and it was found that the normal shock moves closer to the nozzle exit plane as particle loading is increased An empirical correlation of the data is presented which is valid for both the gas only and the gas-particle plumes

The test facility is shown in Fig 1 The system consists of a high-pressure (stagnation pressure approximately equal to 100 psia) gas-only or gas-particle mixture which passes through a nozzle into a low pressure reservoir of 3800 ft$^3$ volume The mixing of the particles and the gas is accomplished by forcing the particles into the gas stream by means of a motor-driven piston In addition to schlieren photography of the plume, instrumentation includes measurements of stagnation, exit plane, and tank pressures and gas and particle flow rates Two different conical nozzles were used Each has a throat diameter of 0 260 in and an ex pansion cone half-angle of 15°, but one has an area ratio (exit to throat) of 1 385 whereas the other is 3 868


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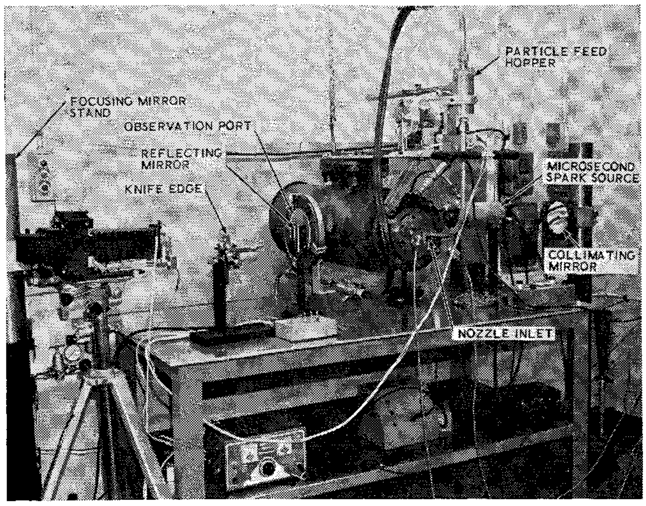


**Fig 1 Gas-particle flow facility**

The flow parameters that were varied consist of the particle-to-gas mass fraction $\phi$, the ratio of the exit plane static pressure to the ambient (tank) pressure $p /p_\infty$, the nozzle exit Mach number $M$ , and the ratio of specific heats $\gamma$ The distance to the normal shock was determined directly from the schlieren photographs Gas-only experiments using nitrogen, carbon dioxide, and helium were made to provide a basis for comparison with the data of the gas-particle runs as well as to determine the effect of $\gamma$ on the distance to the normal shock It was found that the data from these tests for both nozzles and all three gases could be represented by

$$\frac{x_g}{d} = 0\ 69M \left(\frac{\gamma p}{p_\infty}\right)^{1/2} \tag{1}$$

where $x_g$ is the distance to the normal shock for gas-only flows, $d$ is the nozzle exit diameter, and $M$ is the nozzle exit Mach number Some of the experimental data are shown compared with Eq (1) in Fig 2 Equation (1) was also compared with experimental results of several other investigators[1–5] covering a range of $\gamma$ from 1 22 to 1 67, a range of $M$ from 1 to 3 and a range of $p /p_\infty$ from 1 to about 550 Included were 5 different gases for the cold flow tests and fir-

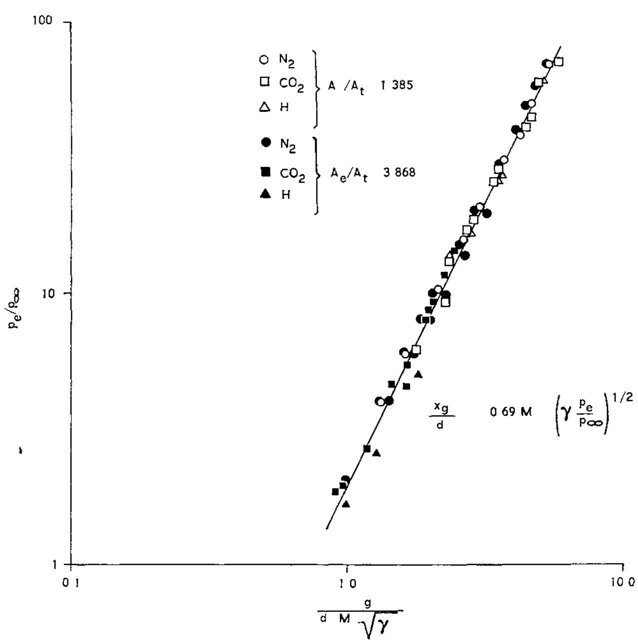


**Fig 2 Distance to the normal shock in gas-only flows normalized by $d$ $M\gamma^{1/2}$**